BENHA UNIVERSITY
FACULTY OF ENGINEERING AT SHOUBRA

ECE-312
Electronic Circuits (A)

Lecture # 7
BJT Low Frequency Response

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Agenda

- Low Frequency Analysis- Bode Plot
- Low Frequency Response – BJT Amplifier with R_L
- Impact of R_S on the BJT Low Frequency Response

LOW FREQUENCY ANALYSIS- BODE PLOT

Defining the Low Cutoff Frequency

- In the low-frequency region of the single-stage BJT or FET amplifier, it is the RC combinations formed by the network capacitors C_C, C_E, and C_s and the network resistive parameters that determine the cutoff frequencies

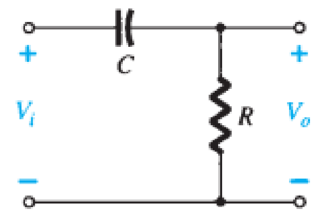


FIG. 9.14
RC combination that will define a low-cutoff frequency.

- Voltage-Divider Bias Config.

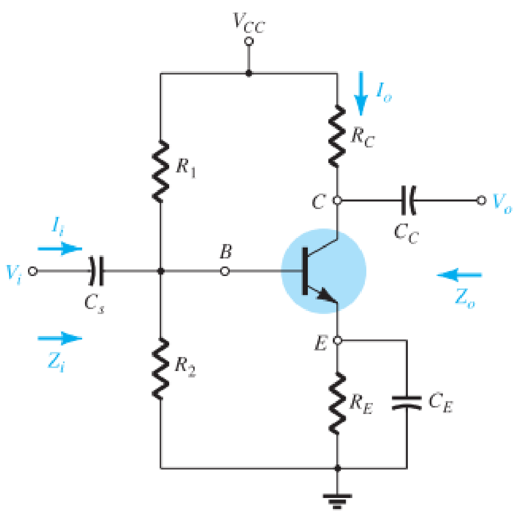


FIG. 9.15
Voltage-divider bias configuration.

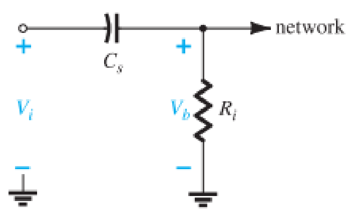


FIG. 9.16
Equivalent input circuit for the network of Fig. 9.15.

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$

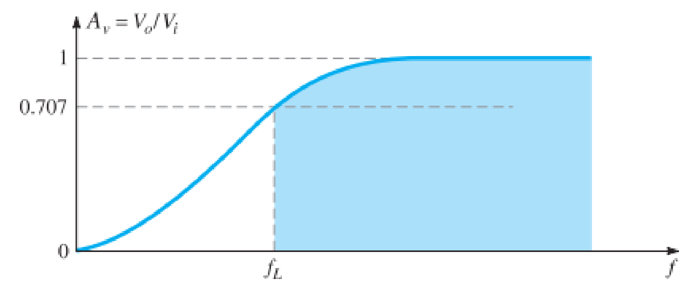
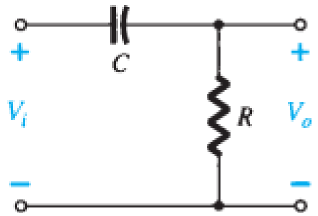


FIG. 9.19
Low-frequency response for the RC circuit of Fig. 9.14.

Defining The Low Cutoff Frequency ..



$$\mathbf{V_o = \frac{RV_i}{R + X_C}}$$

The magnitude of V_o is

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

For the special case where $X_C = R$,

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}}V_i$$

$$|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707|_{X_C=R}$$

$$X_C = \frac{1}{2\pi f_L C} = R$$

$$f_L = \frac{1}{2\pi RC}$$

$$G_v = 20\log_{10} A_v = 20\log_{10}\frac{1}{\sqrt{2}} = -3\text{ dB}$$

$A_v = V_o/V_i = 1$ or $V_o = V_i$ (the maximum value),

$$G_v = 20\log_{10} 1 = 20(0) = 0\text{ dB}$$

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

$$f_L = \frac{1}{2\pi RC}$$

$$A_v = \frac{1}{1 - j(f_L/f)}$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_L/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_L/f)}_{\text{phase} \angle \text{ by which } V_o \text{ leads } V_i}$$

when $f = f_L$,

$$|A_v| = \frac{1}{\sqrt{1 + (1)^2}} = \frac{1}{\sqrt{2}} = 0.707 \Rightarrow -3\text{ dB}$$

$$A_{v(\text{dB})} = 20\log_{10}\frac{1}{\sqrt{1 + (f_L/f)^2}}$$

Bode Plot

$$A_{v(\mathrm{dB})} = 20\log_{10}\frac{1}{\sqrt{1+(f_L/f)^2}}$$

$$A_{v(\mathrm{dB})} = -20\log_{10}\left[1+\left(\frac{f_L}{f}\right)^2\right]^{1/2}$$
$$= -\left(\tfrac{1}{2}\right)(20)\log_{10}\left[1+\left(\frac{f_L}{f}\right)^2\right]$$
$$= -10\log_{10}\left[1+\left(\frac{f_L}{f}\right)^2\right]$$

For frequencies where $f \ll f_L$ or $(f_L/f)^2 \gg 1$,

$$A_{v(\mathrm{dB})} = -10\log_{10}\left(\frac{f_L}{f}\right)^2$$

$$A_{v(\mathrm{dB})} = -20\log_{10}\frac{f_L}{f} \qquad f \ll f_L$$

$$\text{At } f = f_L:\ \frac{f_L}{f} = 1 \text{ and } -20\log_{10}1 = 0\ \mathrm{dB}$$

$$\text{At } f = \tfrac{1}{2}f_L:\ \frac{f_L}{f} = 2 \text{ and } -20\log_{10}2 \cong -6\ \mathrm{dB}$$

$$\text{At } f = \tfrac{1}{4}f_L:\ \frac{f_L}{f} = 4 \text{ and } -20\log_{10}4 \cong -12\ \mathrm{dB}$$

$$\text{At } f = \tfrac{1}{10}f_L:\ \frac{f_L}{f} = 10 \text{ and } -20\log_{10}10 = -20\ \mathrm{dB}$$

- The piecewise linear plot of the asymptotes and associated breakpoints is called a Bode plot of the magnitude versus frequency.

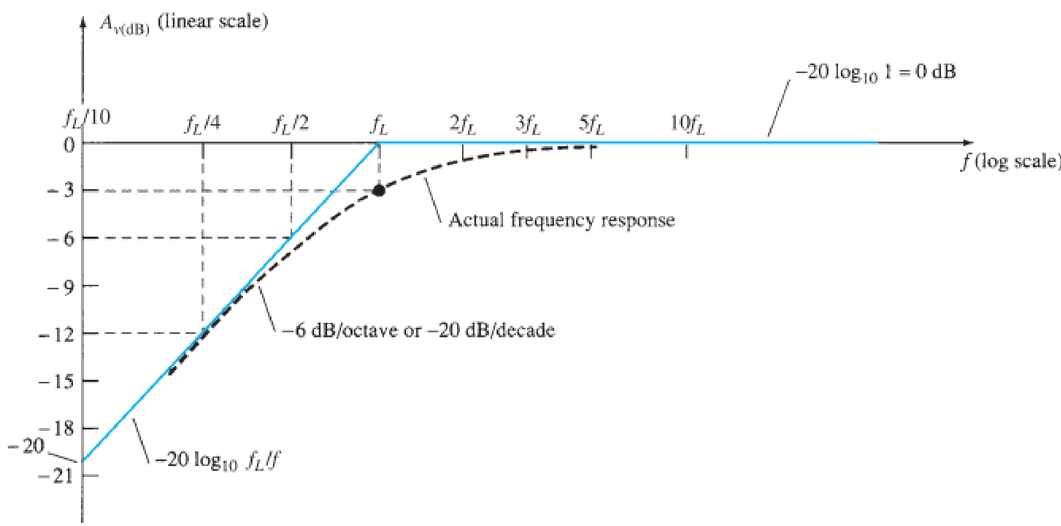


FIG. 9.20
Bode plot for the low-frequency region.

Bode Plot..

- A change in frequency by a factor of two, equivalent to one octave, results in a 6-dB change in the ratio, as shown by the change in gain from $f_L/2$ to f_L.
- For a 10:1 change in frequency, equivalent to one decade, there is a 20-dB change in the ratio, as demonstrated between the frequencies of $f_L/10$ and f_L.

$$A_{v(\text{dB})} = 20 \log_{10} \frac{V_o}{V_i}$$

$$\frac{A_{v(\text{dB})}}{20} = \log_{10} \frac{V_o}{V_i}$$

$$A_v = \frac{V_o}{V_i} = 10^{A_{v(\text{dB})}/20}$$

- Phase Angle:

$$\theta = \tan^{-1} \frac{f_L}{f}$$

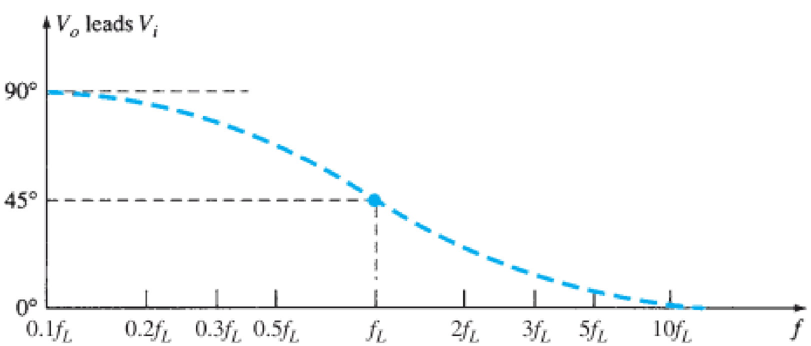


FIG. 9.22
Phase response for the RC circuit of Fig. 9.14.

For frequencies $f \ll f_L$,

$$\theta = \tan^{-1} \frac{f_L}{f} \rightarrow 90^\circ$$

For instance, if $f_L = 100f$,

$$\theta = \tan^{-1} \frac{f_L}{f} = \tan^{-1}(100) = 89.4^\circ$$

For $f = f_L$,

$$\theta = \tan^{-1} \frac{f_L}{f} = \tan^{-1} 1 = 45^\circ$$

For $f \gg f_L$,

$$\theta = \tan^{-1} \frac{f_L}{f} \rightarrow 0^\circ$$

For instance, if $f = 100f_L$,

$$\theta = \tan^{-1} \frac{f_L}{f} = \tan^{-1} 0.01 = 0.573^\circ$$

Example

EXAMPLE 9.10 For the network of Fig. 9.23:

a. Determine the break frequency.
b. Sketch the asymptotes and locate the −3-dB point.
c. Sketch the frequency response curve.
d. Find the gain at $A_{v(\mathrm{dB})} = -6$ dB.

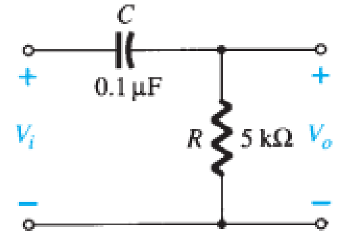


FIG. 9.23
Example 9.10.

Solution:

a. $f_L = \dfrac{1}{2\pi RC} = \dfrac{1}{(6.28)(5 \times 10^3\ \Omega)(0.1 \times 10^{-6}\ \mathrm{F})}$

$\cong$ **318.5 Hz**

b. and c. See Fig. 9.24.

d. Eq. (9.27): $A_v = \dfrac{V_o}{V_i} = 10^{A_{v(\mathrm{dB})}/20}$

$= 10^{(-6/20)} = 10^{-0.3} = 0.501$

and $V_o = 0.501\ V_i$ or approximately 50% of V_i.

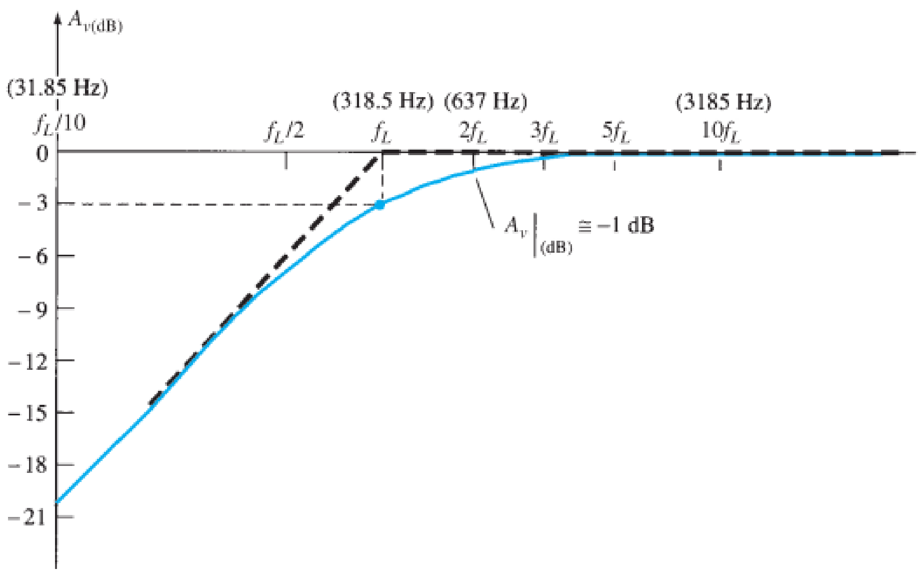


FIG. 9.24
Frequency response for the RC circuit of Fig. 9.23.

LOW FREQUENCY RESPONSE – BJT AMPLIFIER WITH R_L

Loaded BJT Amplifier

In the voltage-divider ct.
→ the capacitors Cs, C_C , and C_E will determine the low-frequency response.

f_L= min(f_{Ls}, f_{Lc}, f_{LE})

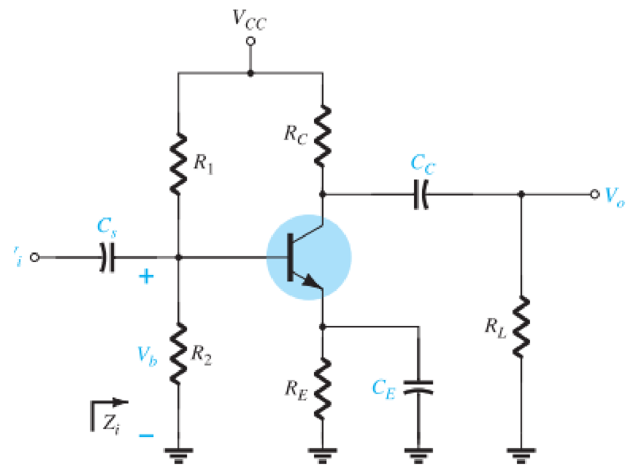


FIG. 9.25
Loaded BJT amplifier with capacitors that affect the low-frequency response.

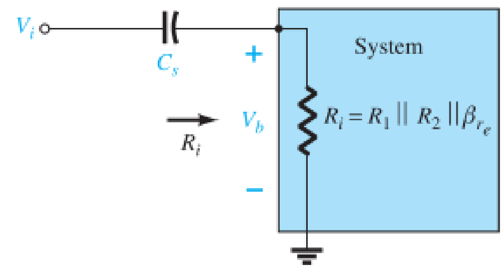


FIG. 9.26
Determining the effect of C_s on the low-frequency response.

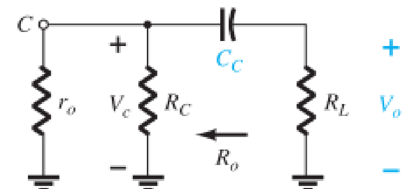


FIG. 9.28
Localized ac equivalent for C_C with $V_i = 0$ V.

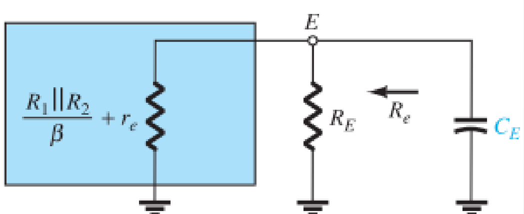


FIG. 9.30
Localized ac equivalent of C_E.

→Cs:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_i}{R_i - jX_{C_s}}$$

$$f_{L_s} = \frac{1}{2\pi R_i C_s}$$

$$R_i = R_1 \| R_2 \| \beta r_e.$$

$$\mathbf{A}_v = \frac{\mathbf{V}_b}{\mathbf{V}_i} = \frac{1}{1 - j(f_{L_s}/f)}$$

→ Cc:

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

$$R_o = R_C \| r_o$$

→ C_E:

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e\right)$$

IMPACT OF R_S ON THE BJT LOW FREQUENCY RESPONSE

Impact of R_S

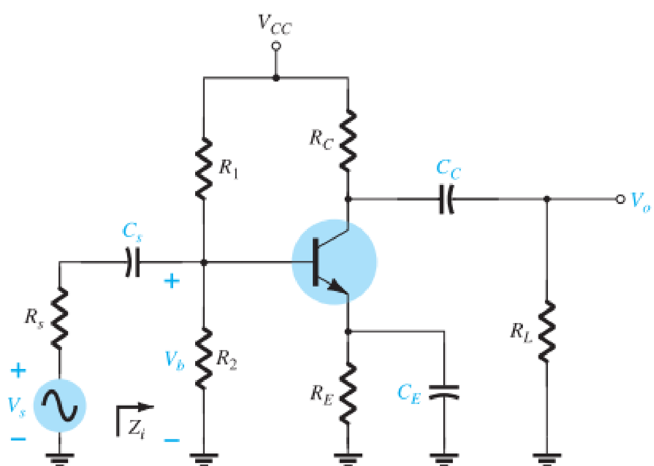


FIG. 9.32
Determining the effect of R_s on the low-frequency response of a BJT amplifier.

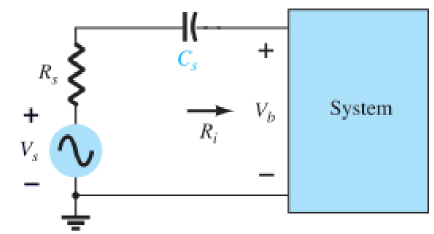


FIG. 9.33
Determining the effect of C_s on the low-frequency response.

$$f_{L_s} = \frac{1}{2\pi(R_i + R_s)C_s}$$

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$R_e = R_E \| \left(\frac{R'_s}{\beta} + r_e\right) \text{ and } R'_s = R_s \| R_1 \| R_2$$

Example

EXAMPLE 9.12

a. Repeat the analysis of Example 9.11 but with a source resistance R_s of 1 kΩ. The gain of interest will now be V_o/V_s rather than V_o/V_i. Compare results.
b. Sketch the frequency response using a Bode plot.
c. Verify the results using PSpice.

Solution: a. The dc conditions remain the same:

$$r_e = 15.76\ \Omega \text{ and } \beta r_e = 1.576\ \text{k}\Omega$$

Midband Gain $A_v = \dfrac{V_o}{V_i} = \dfrac{-R_C \| R_L}{r_e} \cong -90$ as before

The input impedance is given by

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$

and from Fig. 9.35,

$$V_b = \frac{R_i V_s}{R_i + R_s}$$

or

$$\frac{V_b}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\ \text{k}\Omega}{1.32\ \text{k}\Omega + 1\ \text{k}\Omega} = 0.569$$

so that

$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_b}{V_s} = (-90)(0.569) = \mathbf{-51.21}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$

$$f_{L_S} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1\ \text{k}\Omega + 1.32\ \text{k}\Omega)(10\ \mu\text{F})}$$

$$f_{L_S} \cong 6.86\ \text{Hz vs. } 12.06\ \text{Hz without } R_s$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C} = \frac{1}{(6.28)(4\ \text{k}\Omega + 2.2\ \text{k}\Omega)(1\ \mu\text{F})} \cong \mathbf{25.68\ Hz} \text{ as before}$$

C_E

$$R_s' = R_s \| R_1 \| R_2 = 1\ \text{k}\Omega \| 40\ \text{k}\Omega \| 10\ \text{k}\Omega \cong 0.889\ \text{k}\Omega$$

$$R_e = R_E \left\| \left(\frac{R_s'}{\beta} + r_e\right) = 2\ \text{k}\Omega \right\| \left(\frac{0.889\ \text{k}\Omega}{100} + 15.76\ \Omega\right)$$

$$= 2\ \text{k}\Omega \| (8.89\ \Omega + 15.76\ \Omega) = 2\ \text{k}\Omega \| 24.65\ \Omega \cong 24.35\ \Omega$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{3058.36}$$

$$\cong \mathbf{327\ Hz} \text{ vs. } 87.13\ \text{Hz without } R_s.$$

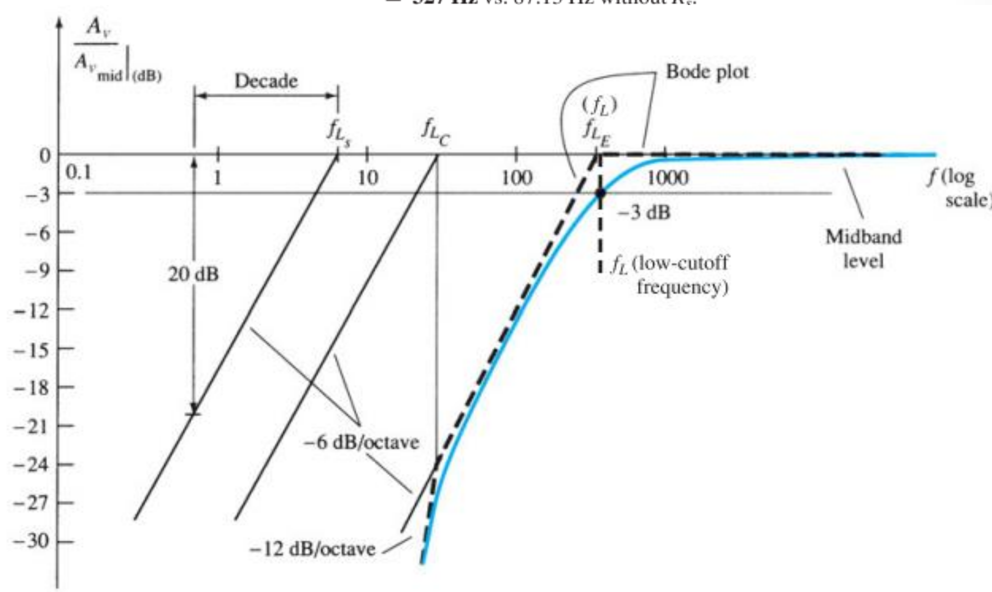


FIG. 9.36
Low-frequency plot for the network of Example 9.12.

- For more details, refer to:
  - Chapter 9 at R. Boylestad, **Electronic Devices and Circuit Theory**, 11th edition, Prentice Hall.
- The lecture is available online at:
  - http://bu.edu.eg/staff/ahmad.elbanna-courses/11966
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg